PREPARATION WORK FOR AP CALCULUS

1. READ THE DIRECTIONS CAREFULLY.

2. FOLLOW THE DIRECTIONS!

3. This work is to be done as a project. This means that **NEATNESS** and **COMPLETENESS** are priorities.
a) All graphs are to be done on graph paper. If you don't have graph paper, go to
http://www.printfreegraphpaper.com

b) Any straight lines should be drawn with a straight edge.
c) If a graphing calculator is used for answers, explain what was done on the calculator. For example, if you found an answer by finding sin 0.7, don't just write: 0.644. . . . You should write sin 0.7 = 0.644 . . .
d) Any approximate answers should be given to three decimal places. Examples: If the exact answer is $^2/_3$, acceptable answers are: $^2/_3$, 0.667 (round to 3 decimal places), 0.666 (truncated to three decimal places), $0.\overline{6}$ (an equivalent answer), 0.6666 (truncated to more than 3 decimal places), 0.66667 (rounded to more than 3 decimal places). Unacceptable answers are 0.6, 0.7, 0.66, 0.67 (all do not have 3 decimal places) and 0.645 (not the correct answer to three decimal places).

4. Any work on the problem pages will NOT be counted nor graded.

5. Each section of problems should be attached to the front of the pages of your work **on just those problems**. For maximum credit, you should have **5 sets** of papers. NOTE: sections 1.2, 1.3 and 1.6 are two pages and there is no section 1.4.

6. I will collect (in random order) one set of problems on each of the first five days of class. You will have five 20 point quizzes (these must be taken outside of your AP Calculus period), one for each section.These five quizzes must be completed by the beginning of your class period on the 7th day of classes. Together, these five quizzes will count as one 100 point test.

7. IF THERE ARE PROBLEMS YOU CANNOT DO, SEEK PROFESSIONAL HELP. Refer back to your Precalculus notes, talk to a classmate and / or email me at duanemiller@wowway.com for help.

8. If you email me with your email address, I will send you the answers to the summer packet problems, one section per week, starting the week of July 21.

9. **NC** means no calculator.

Why you should do the summer packet.

Each section will earn up to 3% extra credit for a total maximum extra credit of (**NC.**)15%. This percent will go into a bank. You may withdraw no more than 3% at the end of any quarter and no more than 10% at

the end of the first semester final exam to increase your grade for that quarter or final exam. Your total withdrawal for the year cannot exceed the percent you earned for your packet. No interest will be earned for your percent in the bank. This will be only one of a few extra credit opportunities.

Section 1.1

Let L be the line determined by points A and B. For exercises 1 & 2: (a) plot A and B; (b) find the slope of L, (c) draw the graph of L and (d) write the equation of L

1. $A(1,-2)$ $B(2,1)$
2. $A(2,3)$ $B(-1,3)$

In exercises 3 & 4, write an equation for (a) the vertical line and (b) the horizontal line through the point P.

3. $P(2,3)$
4. $P\ \ 0,-\sqrt{2}$

In exercises 5 & 6, write the equation in *point-slope form* for the line through the point P with slope m.

5. $P(1,2)$ $m=3$
6. $P(0,3)$ $m=2$

In exercises 7 – 9, write an equation for the line through the two points.

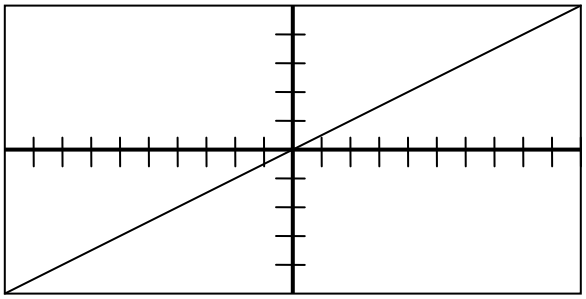
7. (2, 3) (4, 8)
8. (-2, 3) (-2, 7)
9. (3, 5) (-1, 5)

In exercises 10 & 11, write the equation in *slope-intercept form* for the line with slope m and y-intercept b.

10. $m = 3$ $b = -2$
11. $m = -\frac{1}{2}$ $b = -3$

In exercises 12 & 13, the line contains the origin and the point in the upper right corner of the graph. Write an equation for the line.

12.



[-10, 10] by [-25, 25]

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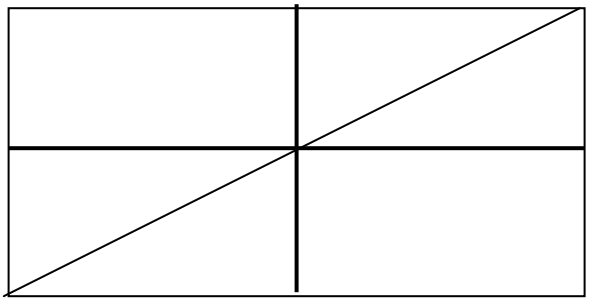

[-15, 15] by [-10, 10]

In exercise 14 – 18, write an equation for the line through P that is (a) parallel to L and (b) perpendicular (or normal) to L.

14. $P(0,0)$ $L: y=-x+2$
15. $P(-2,3)$ $L: 2x+y=4$
16. $P(-2,4)$ $L: x=5$
17. $P(-1,0.5)$ $L: y=3$
18. P (4, 6) L: $y - 2 = 4(x + 5)$

Section 1.2

NC. Solve for x and graph the solution.

1. $3x-1 \le 5x+3$ 2. $|x-3| \ge 4$

3. $x^2 < 16$

Describe how the graph of f can be transformed to the graph of g.

4. $f(x) = x^2$ $g(x) = (x+2)^2 - 3$

5. $f(x) = |x|$ $g(x) = |x-5| + 2$

In exercises 6 – 9, write a formula that expresses the first variable as a function of the second variable.

Example: Write the surface area (S) of a cube as a function of its volume (V). Solution: The volume of a cube is $V = s^3$ (where s is the length of an edge of a cube), so $s = \sqrt[3]{V}$. The surface area of a cube is $S = 6s^2$.

Substituting for s, we get $S = 6\left(\sqrt[3]{V}\right)^2$ which expresses S as a function (or in terms) of V.

6. the area (A) of a circle as a function of its diameter (d).
7. the height (h) of an equilateral triangle as a function of its side length (s).
8. the radius of a sphere (r) as a function of the sphere's volume (V).
9. the area of a square (A) as a function of the length of its diagonal (d).

For problems 10 – 23, (a) find the domain, (b) find the range, (c) graph the function and (d) determine if the function is symmetric with respect to the y-axis, origin or neither.

10. $y = 4 - x^2$ 11. $y = x^2 - 9$

12. $y = 2 + \sqrt{x-1}$ 13. $y = -\sqrt{-x}$

14. $y = 2\sqrt{3-x}$ 15. $y = \dfrac{1}{x-2}$

16. $y = \sqrt[3]{x-3}$ 17. $y = \sqrt[3]{1-x^2}$

18. $y = \sqrt[4]{-x}$ 19. $y = 1 + \dfrac{1}{x}$

20. $y = \sqrt{4-x^2}$ 21. $y = x^{2/3}$

22. $y = 1 + \dfrac{1}{x^2}$ 23. $y = x^{3/2}$

Determine whether the function is even, odd or neither. Do 5 of the problems graphically and the remaining 5 problems analytically (using algebra!).

24. $y = x^4$ 25. $y = x + x^2$

26. $y = x + 2$ 27. $y = x^2 - 3$

28. $y = \sqrt{x^2+2}$ 29. $y = x + x^3$

30. $y = \dfrac{x^3}{x^2-1}$ 31. $y = \sqrt[3]{2-x}$

32. $y = \dfrac{1}{x-1}$ 33. $y = \dfrac{1}{x^2-1}$

In exercises 34 – 38, (a) draw the graph of the function, find the (b) domain and (c) the range.

34. $f(x) = -|3-x| + 2$

35. $f(x) = 2|x+4| - 3$

36. $f(x) = \begin{cases} 3-x, & x \le 1 \\ 2x, & x > 1 \end{cases}$

37. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$

38. $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x-1, & x > 1 \end{cases}$

In exercises 39 – 42, use the vertical line test to determine whether the curve is the graph of a function.

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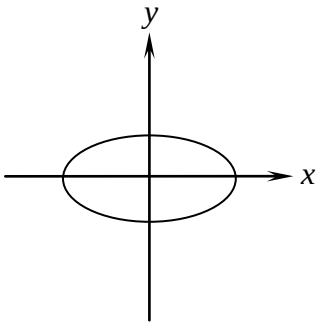

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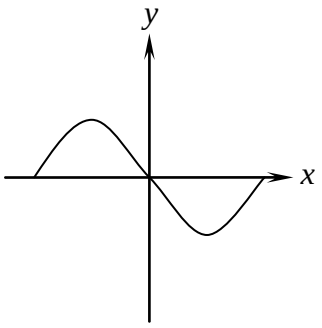

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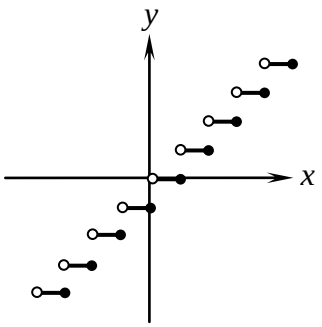

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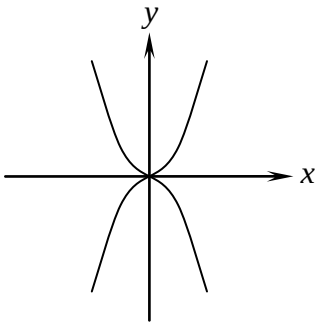

In exercises 43 – 46, write a piecewise formula for the function. Your answers should look like problems 36 – 38 from this section.

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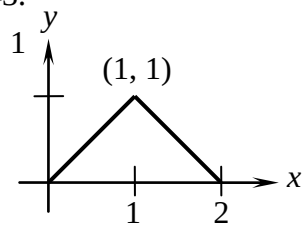

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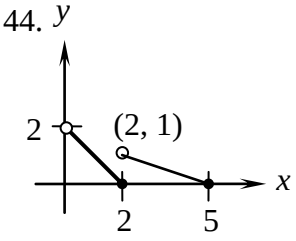

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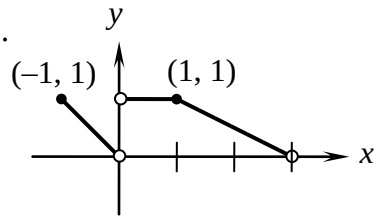

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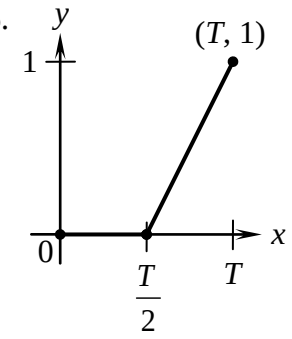


In exercises 47 and 48, find

(a) $f(3)$ (b) $g(-3)$ (c) $f \quad g(0)$

(d) $g \quad f(0)$ (e) $g \quad g(-2)$ (f) $f \quad g(x)$

(g) $g \quad f(x)$ (h) $f \quad f(x)$

47. $f(x) = x+1, \quad g(x) = x-1$

48. $f(x) = x+5, \quad g(x) = x^2 - 3$

Section 1.3

Evaluate the expression. Round your answers to three (3) decimal places.

1. $5^{2/3}$ 2. $3^{\sqrt{2}}$ 3. $3^{-1.5}$

Solve the equation. Round your answers to three (3) decimal places.

4. $x^3 = 17$ 5. $x^5 = 24$ 6. $x^{10} = 1.4567$

Simplify the exponential expression. (You should have at most of each base and no negative exponents.)

7. $\dfrac{x^{-3}y^{2}}{x^{4}y^{3}}^{2}{}^{3}$ 8. $\left(\dfrac{a^3b^{-2}}{c^4}\right)^2\left(\dfrac{a^4c^{-2}}{b^3}\right)^{-1}$

Graph the function. State its domain, range and intercepts.

9. $y = -2^x + 3$ 10. $y = e^x + 3$

11. $y = 3e^{-x} - 2$ 12. $y = -2^{-x} - 1$

Rewrite the exponential expression to have the indicated base.

13. 9^{2x}, base 3 14. 16^{3x}, base 2

15. $\left(\dfrac{1}{8}\right)^{2x}$, base 2 16. $\left(\dfrac{1}{27}\right)^{x}$, base 3

Use graphs to solve the equations. Answers should be exact to 3 decimal places.

17. $2^x = 5$ 18. $e^x = 4$

19. $3^x - 0.5 = 0$ 20. $3 - 2^{-x} = 0$

NC. In exercises 21 – 26, match the function with its graph (a) – (f). The graphs are from a TI-86 with a ZSTD screen. Press CLEAR to remove the menu from your screen.

21. $y = 2^x$

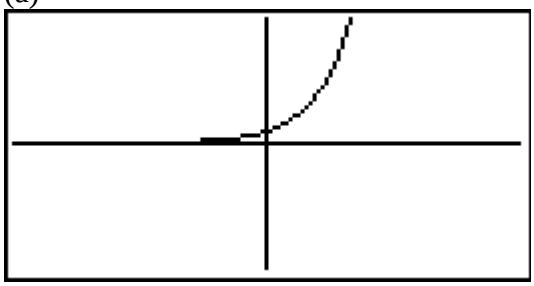
22. $y = 3^{-x}$

23. $y = -3^{-x}$

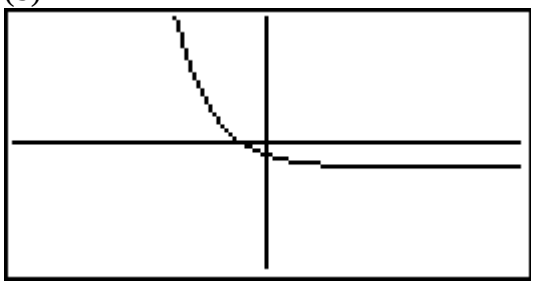
24. $y = 2^{-x} - 2$

25. $y = 1.5^x - 2$

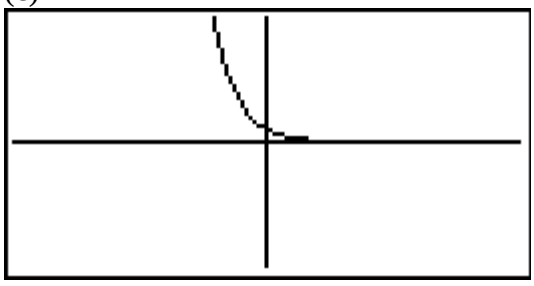
(a)



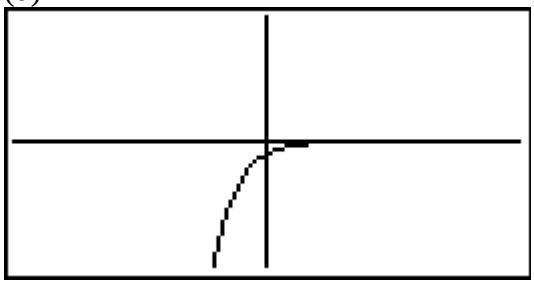
(b)



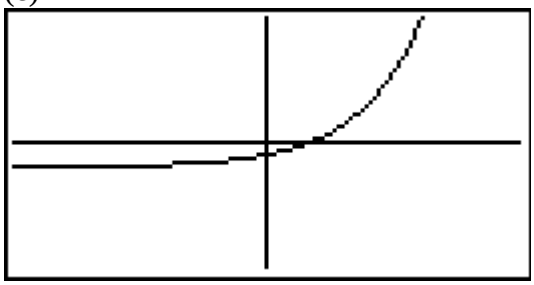
(c)



(d)



(e)



Section 1.5

Let $f(x) = \sqrt[3]{x-1}$, $g(x) = x^2 + 1$ and evaluate the expression.

1. $f \circ g$ (1)
2. $g \circ f$ (–7)
3. $f \circ g$ (x)
4. $g \circ f$ (x)

Determine the intersection of the two curves. Round your answer to 3 decimal places.

5. $y = 2x - 3$, $y = 5$
6. $y = -3x + 5$, $y = -3$
7. $y = 2^x$, $y = 3$
8. $y = \log(-x)$, $y = -1.234$
9. $y = e^{-x}$, $y = 4$
10. $y = \ln(-x)$, $y = -5.678$

Determine whether the function is one to one.

11. $y = 2|x|$
12. $y = x + 1$
13. $y = \dfrac{1}{x}$
14. $y = \dfrac{1}{x^2 + 1}$
15. $y = -2x^3$
16. $y = \lfloor x \rfloor$ or $y = \text{int}\, x$

Determine whether the function has an inverse function. (Use the same test you used in problems 11 – 16!)

17. $y = \dfrac{3}{x-2} - 1$
18. $y = x^2 + 5x$
18. $y = x^3 - 4x + 6$
20. $y = x^3 + x$
21. $y = \ln x^2$
22. $y = 2^{3-x}$

For problems 23-28, find the inverse of f, denoted f^{-1}. Verify for #23-25 that $f\left(f^{-1}(x)\right) = x$. Verify for #26-28 that. $f^{-1}\left(f(x)\right) = x$

23. $y = 2x + 3$
24. $y = x^3 - 1$
25. $y = x^2$, $x \le 0$
26. $y = -|x-2|^2$, $x \le 2$
27. $y = x^2 + 2x + 1$, $x \ge -1$
28. $y = \dfrac{2x+1}{x+3}$

29. The domain is all real numbers in problems 23, 24 and 28. The domain is restricted in problems 25, 26 and 27 as indicated by the inequality given as part of those problems. Explain why the restriction is necessary.

Section 1.6

Convert from radians to degrees or degrees to radians.

1. $\pi/3$ 2. –2.5

3. –40° 4. 45°

Solve the equation graphically in the given interval. (RADIAN mode!)

5. $\sin x = 0.6, \quad 0 \le x < 2\pi$

6. $\cos x = -0.4, \quad 0 \le x < 2\pi$

7. $\tan x = 1, \quad -\frac{\pi}{2} \le x < \frac{3\pi}{2}$

8. Show <u>analytically</u> that $f(x) = 2x^2 - 3$ is an even function. Explain why its graph is symmetric with respect to (wrt) the *y*-axis.

9. Show <u>analytically</u> that $f(x) = x^3 - 3x$ is an odd function. Explain why its graph is symmetric with respect to (wrt) the origin.

In exercises 10 – 13, the angle lies at the center of a circle and subtends an arc of the circle. Find the missing angle measure, circle radius or arc length.

	Angle	Radius	Arc Length
10.	$5\pi/8$	2	x
11.	175°	y	10
12.	z	14	7
13.	w	6	$32/8$

In exercises 14 – 19, choose an appropriate viewing window to display two complete periods of each trigonometric function (RADIAN mode!)

14. $y = \sec x$ 15. $y = \csc x$ 16. $y = \cot x$

17. $y = \sin x$ 18. $y = \cos x$ 19. $y = \tan x$

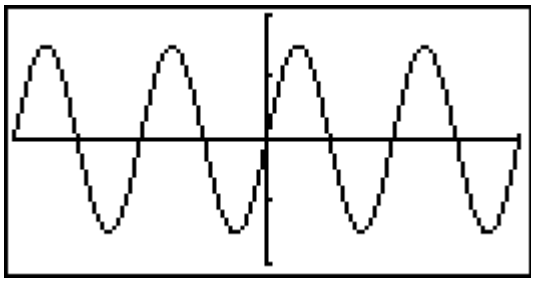
In exercises 20 – 23, give the measure of the angle (RADIAN mode!). Give exact answers when possible. (i.e. if the given measure is on your unit circle, you should give the angle in terms of π.)

20. $\sin^{-1}(0.5)$ 21. $\cos^{-1} -\frac{\sqrt{2}}{2}$

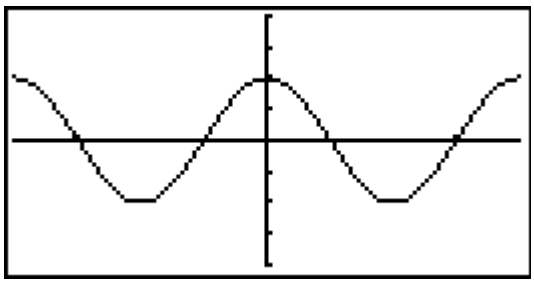
22. $\tan^{-1}(-5)$ 23. $\cos^{-1}(0.7)$

In exercises 24 – 29, specify the (a) period, (b) amplitude and (c) the viewing window that is shown. The graphs are from a TI-86 with a ZSTD screen. Press CLEAR to remove the menu from your screen.

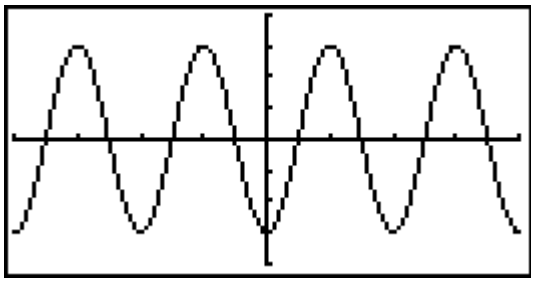
24. $y = 1.5\sin\ 2x$



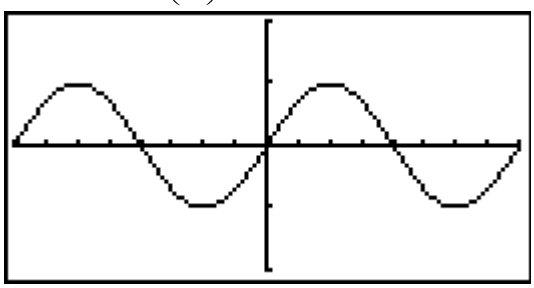
25. $y = 2\cos\ 3x$



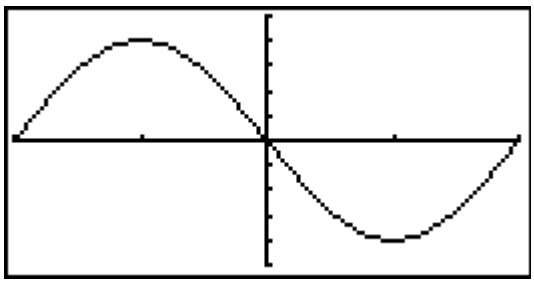
26. $y = -3\cos\ 2x$



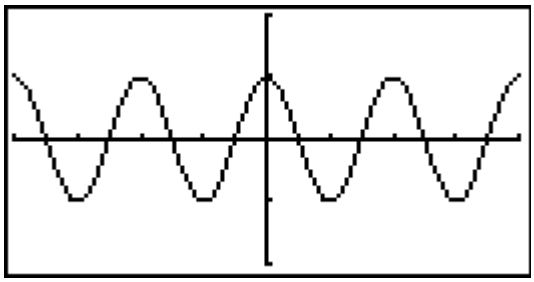
27. $y = 5\sin\left(\frac{x}{2}\right)$



28. $y = -4\sin\left(\frac{\pi}{3}x\right)$



29. $y = \cos\ \pi x$



NC. In exercises 30 – 33, use the given information to find the <u>exact value</u> of the six trigonometric functions at the angle θ.

30. $\sin^{-1}\left(\frac{8}{17}\right)$ 31. $\tan^{-1}\left(-\frac{5}{12}\right)$

32. The point $P(3,-4)$ is on the terminal side of θ.

33. The point $P(-2,2)$ is on the terminal side of θ.

NC. In exercises 34 and 35, give the exact answer to the expression.

34. $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$ 35. $\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right)$

36. NC. Fill in the table below with exact values. you may leave radicals in the denominator.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$					
$\cos\theta$					
$\tan\theta$					
$\cot\theta$					
$\sec\theta$					
$\csc\theta$					